What a Drag! Accommodating Assumptions

*Calculating Position with Drag Educators Supplement*

The “Calculating Position with Drag” worksheet is given in 3 pages and should be printed back to back. The third page on the student copy is actually the first page. It contains the initial conditions and is easy to separate from the table data after printing.

The first step to completing the worksheet is filling in the initial conditions. Below is each condition with data entered (or a source to find local data).

Acceleration along the x-axis: $\frac{dV\_{x}}{dt}=-\frac{\left(\frac{1}{2}pV^{2}\cos(θ)∙C\_{d}S\right)}{m}$

Drag equation ÷ mass = ft/sec which is what we need. Drag alone is a measure of force.

Acceleration along the y-axis: $\frac{dV\_{y}}{dt}=-\frac{\left(\frac{1}{2}pV^{2}\sin(θ)∙C\_{d}S\right)}{m}-g$

Same as above, however acceleration along the y-axis is also affected by gravity.

$p=$ Atmospheric density = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ slugs/ft3

Visit <http://www.digitaldutch.com/atmoscalc/> to calculate atmospheric density based on the altitude and temperature of your specific area.

V=Velocity = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ ft/sec

Choose an initial velocity for all students to use (around 70fps)

$θ=$Angle of attack = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ degrees

Choose an initial angle of attack (around 80o)

$C\_{d}=$ Drag coefficient = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

We experimentally calculated this value to be about 0.31 for a 2-liter soda bottle

S = Cross sectional area = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ ft2

For a standard 2-liter bottle this value is .0985

W = Weight of object = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ lbs

Choose an initial value (around 0.23 lbs)

g = Gravitational constant = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ ft/sec2

32 ft/sec2

m = Mass of object = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ slugs

Calculate using the formula W ÷ g = m

As the object moves through the air the angle of attack and the velocity will change due to drag and gravity. This makes it impossible to predict the outcome based solely on the input data. Instead, changes in angle and velocity must be considered in very small segments of time.

This is an opportunity to discuss the path that the rocket will follow when drag is included in the calculation. A good way to introduce this is to consider each path the rocket takes if the velocity changes due to drag are taken into account at different time intervals.

1 second intervals: vs. 0.5 second intervals:

Though we are given an initial velocity, we will need to understand the horizontal and vertical components of that velocity as they relate to the angle of attack ($θ$).

This should be a simple trigonometry problem for students and many will come up with the vertical and horizontal components of velocity on their own.

VerticalVelocity = Velocity $∙\sin(\left(θ\right))$

HorizontalVelocity = Velocity $∙\cos(\left(θ\right))$

The table below describes how to find the first three rows of the table data for the student worksheet. The first row will be filled in with all the initial conditions however the last two calculations will require the use of the formula from the first page. For each column you can see the calculation that should take place. Consider V0 as the initial V and V1 as V from the next timestep and so on. Also $∆t$ = 0.2 s throughout the problem as that is our timestep. It is important to highlight the connection between the acceleration equation, the change in velocity, and the change in position for the system.

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Time | Horizontal Velocity (Vx) | Vertical Velocity (Vy) | Velocity (V) | $$θ$$ | Horizontal Position (x) | Vertical Position (y) | $$\frac{dV\_{x}}{dt}$$ | $$\frac{dV\_{y}}{dt}$$ |
| 0.0 | $$V∙\cos(\left(θ\right))$$ | $$V∙sin⁡(θ)$$ | Initial Velocity from first page | $tan^{-1}\left(\frac{V\_{y0}}{V\_{x0}}\right)$ should be 80o | 0 | 0 | Formula from first page with data from this row plugged in along with initial conditions. | Formula from first page with data from this row plugged in along with initial conditions. |
| 0.2 | $$V\_{x0}+\frac{dV\_{x}}{dt}\*∆t$$The velocity from the above cell + the acceleration from the previous timestep\*.02 to arrive at new velocity. | $$V\_{y0}+\frac{dV\_{y}}{dt}\*∆t$$The velocity from the above cell + the acceleration from the previous timestep \* .02 to arrive at new velocity. | $$\sqrt{V\_{x1}^{2}+V\_{y1}^{2}}$$Recalculation of V for use in other equations. | $$tan^{-1}\left(\frac{V\_{y1}}{V\_{x1}}\right)$$ | $$x\_{0}+V\_{x1}∙∆t$$New position is the position from the above cell + the horizontal velocity\*.02 | $$y\_{0}+V\_{y1}∙∆t$$New position is the position from the above cell + the vertical velocity\*.02 | Formula from first page with data from this row plugged in. | Formula from first page with data from this row plugged in. |
| 0.4 | $$V\_{x1}+\frac{dV\_{x}}{dt}\*∆t$$ | $$V\_{y1}+\frac{dV\_{y}}{dt}\*∆t$$ | $$\sqrt{V\_{x2}^{2}+V\_{y2}^{2}}$$ | $$tan^{-1}\left(\frac{V\_{y2}}{V\_{x2}}\right)$$ | $$x\_{1}+V\_{x2}∙∆t$$ | $$y\_{1}+V\_{y2}∙∆t$$ | Formula from first page with data from this row plugged in. | Formula from first page with data from this row plugged in. |

Students may notice that certain values from the acceleration formulas do not change from one row to another. Encourage them to find ways to complete the table faster using the constant values as a single term.